

STANDARD 3: PATTERNS, ALGEBRA AND FUNCTIONS

Students use algebraic methods to explore, model and describe patterns, relationships and functions involving numbers, shapes, data and graphs within a variety of real-world problem-solving situations.

READINESS (Kindergarten)

Students know and are able to do the following:

- **3M-R1. Create, describe and extend a variety of patterns, using concrete objects**
- **3M-R2. Recognize that the same patterns can emerge from a variety of manipulatives and real-world situations**

FOUNDATIONS (Grades 1-3)

Students know and are able to do all of the above and the following:

- **3M-F1. Create, describe and extend a variety of patterns using shapes, events, designs and numbers**

Note: Types of patterns: manipulatives, symbols, words, numbers and pictures

PO 1. Create a pattern using a model (e.g., symbolically: numbers or letters; visually: shapes, designs, numbers or pictures; auditorially: clapping, singing or listening; and kinesthetically: dancing, movement or tactile)

PO 2. Communicate orally or in written form the repetition of objects in a pattern

PO 3. Communicate orally or in written form a given pattern occurring in a sequence of numbers (e.g., counting by 10's, 5's, 3's, 2's, odd, even, forward and backward)

PO 4. Extend patterns using a model

PO 5. Extend a given pattern occurring in a sequence of numbers

- **3M-F2. Formulate generalizations about patterns (e.g., color, shape, size, direction, orientation) to make predictions**

PO 1. Make predictions based on a given pattern

- **3M-F3. Represent and describe how changing the value of one variable results in a change in another**

PO 1. Describe in a given situation how a change in one variable results in the change of another (e.g., if you have to share a batch of cookies with friends, the more friends you have, the fewer cookies you'll each get)

- **3M-F4. Represent and describe mathematical relationships such as order, grouping, etc. (e.g., given a string of numbers, describe the pattern, define the relationship between the numbers and determine the next number in line)**

PO 1. Identify the pattern in skip counting

PO 2. Determine the next number in a skip counting pattern

- **3M-F5. Recognize the symbols of equality and inequality**

PO 1. Use the symbols $<$, $>$, $=$ to compare whole numbers

- **3M-F6. Find missing elements in number sentences**

PO 1. Find the missing number in addition and subtraction number sentences

ESSENTIALS (Grades 4-8)

Students know and are able to do all of the above and the following:

- **3M-E1. Use algebraic methods (write number sentences, in the form of expressions and equations) to explore, model and describe patterns and functions involving numbers, shapes, data, graphs and data plots**

(Grades 4-5)

PO 1. Extend simple geometric and number patterns (e.g., 1, 1, 2, 1, 1, 3, 1, 1, 4 . . .)

PO 2. Create simple geometric and number patterns

PO 3. Describe a rule for a simple pattern (e.g., 5, 10, 15, 20 . . . rule = add five or count by fives)

(Grades 6-8)

PO 4. Generate patterns using algebraic expressions

- **3M-E2. Describe, represent and analyze patterns and relationships using shapes, tables, graphs, data plots, verbal rules and standard algebraic notation**

Note: This concept is covered in 3M-E1 and 3M-E4

- **3M-E3. Describe the concepts of variables, expressions, equations and inequalities**

(Grades 4-5)

Note: There are no POs at this level

(Grades 6-8)

- PO 1. Describe and use variables in a contextual situation
- PO 2. Evaluate an expression using substitution with four basic operations on whole numbers
- PO 3. Translate a written phrase to an algebraic expression and vice versa (words to symbols and symbols to words) (e.g., the quotient of x and y)
- PO 4. Express a simple inequality from a contextual situation (e.g., Joe earns more than \$5.00 an hour: therefore, $x > 5$)

- **3M-E4. Analyze functional relationships to explain how a change in one variable results in a change in another**

(Grades 4-5)

- PO 1. Describe a real-life situation in which a change in one variable results in the change of the other (e.g., temperature in the classroom goes up and the amount of clothing goes down)
- PO 3. Compute an “output” for a given “input” in a function

(Grades 6-8)

- PO 2. Produce the rule (function) that explains the relationship (pattern) between the numbers when a change in the first variable affects the second variable (T-chart, two-row table, or input/output machine)
- PO 4. Complete a T-chart for a given rule

- **3M-E5. Use patterns and functions to represent and solve problems both formally and informally (e.g., measuring the height a ball bounces by dropping different balls from different starting heights)**

(Grades 4-5)

Note: There are no POs at this level

(Grades 6-8)

- PO 1. Solve a problem given a pattern both formally and informally (e.g., “In a patterned necklace, how many red and green beads do you need for a 20-inch necklace?”)

- **3M-E6. Distinguish between linear and nonlinear functions through investigations**

(Grades 4-5)

Note: There are no POs at this level

(Grades 6-8)

PO 1. Distinguish between linear and nonlinear functions, given graphic examples

- **3M-E7. Solve simple linear equations and inequalities using a variety of methods (e.g., informal, formal, graphical) and a variety of manipulatives**

(Grades 4-5)

PO 1. Solve equations using

A. whole numbers with one variable--one step

PO 3. Graph given data points to represent a linear equation

A. on a coordinate grid with whole numbers

(Grades 6-8)

PO 1. Solve equations using

B. whole numbers with one variable--multiple steps

PO 2. Solve linear (first degree) equations using models/manipulatives, symbols and/or graphing in a one-step equation

PO 3. Graph given data points to represent a linear equation

B. in (x, y) form using all four quadrants of a coordinate grid

- **3M-E8. Develop, analyze and explain methods for solving proportions**

(Grades 4-5)

Note: There are no POs at this level

(Grades 6-8)

PO 1. Describe how to solve a problem in context using a proportion

PO 2. Compare quantities using ratios

PO 3. Solve proportions using formal (e.g., cross product) or informal methods (e.g., diagrams, geometric models)

PROFICIENCY (Grades 9-12)

Students know and are able to do all of the above and the following:

- **3M-P1. Model real-world phenomena (e.g., compound interest or the flight of a ball) using functions and relations (e.g., linear, quadratic, sine and cosine, and exponential)**

Core – will be tested on AIMS

PO 2. Describe a real-world situation that is depicted by a given graph

Beyond Core*

PO 1. Identify the independent and dependent variables from a real-world situation

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 3. Sketch a graph that models a given real-world situation

- **3M-P2. Represent and analyze relationships using written and verbal explanations, tables, equations, graphs and matrices and describe the connections among those representations**

Core – will be tested on AIMS

PO 3. Determine whether a relation is a function, given the graphical representation

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 1. Express the relationship between two variables using a table, equation, graph and matrix

{PO 2 Deleted}

*Beyond Core: Appropriate to be taught after a grounding in core instruction, but will not be tested on AIMS

- **3M-P3. Analyze the effects of parameter changes on functions (e.g., linear, quadratic and trigonometric) using calculators and/or computers**

Beyond Core

PO 1. Use technology to determine changes in the shape and behavior of polynomial functions (of degree 2 or less) when constants and coefficients are varied

- **3M-P4. Interpret algebraic equations and inequalities geometrically and describe geometric relationships algebraically**

Core – will be tested on AIMS

- PO 1. Graph a linear equation in two variables
- PO 2. Graph a linear inequality in two variables
- PO 3. Determine slope and intercepts of a linear equation
- PO 4. Write an equation of the line that passes through two given points
- PO 5. Determine from two linear equations whether the lines are parallel, are perpendicular or coincide

- **3M-P5. Apply trigonometry to real-life problem situations (e.g., investigate how to find the distance across a river using similar triangles and trigonometric ratios; compare the sine and cosine curves to the curves of sound waves)**

Core – to be taught in grades 9-10, but will not be tested on AIMS

- PO 1. Use the definitions of trigonometric functions to find the sine, cosine and tangent of the acute angles of a right triangle

Beyond Core

- PO 2. Solve simple right-triangle trigonometric equations involving sine, cosine and tangent
- PO 3. Use an appropriate right-triangle trigonometric model to solve a real-life problem

- **3M-P6. Perform mathematical operations on expressions and matrices, and solve equations and inequalities**

Core – will be tested on AIMS

- PO 1. Simplify numerical expressions using the order of operations, including exponents
- PO 2. Evaluate algebraic expressions using substitution
- PO 3. Simplify algebraic expressions using distributive property
- PO 4. Simplify square roots and cube roots with monomial radicands that are perfect squares or perfect cubes
- PO 6. Evaluate numerical and algebraic absolute value expressions
- PO 7. Multiply and divide monomial expressions with integer exponents
- PO 9. Solve linear equations and inequalities in one variable
- PO 10. Solve formulas for specified variables
- PO 11. Solve quadratic equations (integral roots only)
- PO 13. Solve proportions which generate linear equations
- PO 15. Solve systems of linear equations in two variables (integral coefficients and solutions)

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 5. Calculate powers and roots of real numbers, both rational and irrational, using technology

PO 14. Solve absolute value equations containing a single absolute value expression

Beyond Core

PO 8. Add, subtract and perform scalar multiplication with matrices

PO 12. Solve radical equations involving one radical (restrict to square roots)

- **3M-P7. Translate among tabular, symbolic and graphical representations of functions**

Core – will be tested on AIMS

PO 1. Create a linear equation from a table of values

PO 2. Create a graph from a table of values

PO 3. Determine the solution to a system of equations in two variables, from a given graph

Core – to be taught in grades 9-10, but will not be tested on AIMS

PO 4. Determine the solution to a system of inequalities in two variables, from a given graph (e.g., “Which of the shaded regions represents the solution to the system?”)

- **3M-P8. Use the power of mathematical abstraction and algebraic symbolism to represent various situations**

Core – will be tested on AIMS

PO 1. Translate verbal expressions and sentences to mathematical expressions and sentences

PO 2. Generate an algebraic sentence to model real-life situations, given a data set (limited to linear relationships)

- **3M-P9. Determine maximum and minimum points of a graph and interpret results in problem situations**

Core – will be tested on AIMS

PO 2. Determine domain and range of a relation, given the graph or a set of points

{PO 1 and PO 3 Deleted}

- **3M-P10. Investigate the limiting process by examining infinite sequences and series and areas under curves**

Beyond Core

- PO 1. Compare the estimates of the area under a curve over a bounded interval, using progressively smaller rectangles (not using calculus)
- PO 2. Estimate the limit of a given infinite sequence (e.g., given the sequence $1/n$, as n gets larger) (not using calculus)

DISTINCTION (Honors)

Students know and are able to do all of the above and the following:

- **3M-D1. Use matrices to solve linear systems**
- **3M-D2. Demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations**
- **3M-D3. Understand operations on, and the general principles and behavior of, classes of functions (including logarithmic functions)**
- **3M-D4. Apply general graphing techniques to trigonometric functions**
- **3M-D5. Solve trigonometric equations and verify trigonometric identities**
- **3M-D6. Understand the connections between trigonometric functions and polar coordinates, complex numbers and series**
- **3M-D7. Understand the conceptual foundations of limits, the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines**
- **3M-D8. Analyze the graphs of polynomial, rational, radical and transcendental functions**

